The Hardy spaces H^p also make their entrance and new names such as Weiss, Coifman, and Strömberg can be added to the list of relevant wavelet people.

Chapters 3 and 4 present time-scale algorithms for signal processing and image processing. Chapters 5–7 deal with time–frequency algorithms. In Chapter 3, quadrature mirror filters are introduced. They are used to decompose a signal f, sampled on a fine grid, into a trend and a sequence of fluctuations. The inverse transformation is calculated by the reconstruction property of quadrature mirror filters. The asymptotic behavior of the algorithm is given if the sampling grid becomes infinitely fine. Chapter 4 describes the pyramid algorithms of Burt and Adelson and other orthogonal and biorthogonal pyramid algorithms that calculate iteratively approximations of a given signal at different scales. The Gabor wavelets, treated in Chapter 5, introduced the search for a representation of a signal in time–frequency atoms. To this end, the Wigner–Ville transform is described, but this transform unfortunately did not lead to an algorithm for atomic decomposition. Gabor wavelets give an optimal localization in the time–frequency plane. The Malvar wavelets (Chapter 6) and the wavelet packets (Chapter 7) do not have this optimality, but they give us a whole universe of interesting orthonormal bases. The author then suggests an entropy criterion to select the algorithm and the wavelets that give an optimal decomposition.

In Chapter 8, the coding of an image using the zero-crossings of the wavelet transform is explained. Chapter 9 gives an application in the determination of the fractal exponent of the Weierstrass function (Riemann's example of a continuous function which is nowhere differentiable). Chapters 10 and 11 are not worked out in as much detail as the previous chapters and pose some challenges and open problems concerning the multifractal approach of turbulence and the hierarchical organization of galaxies and the structure of the universe.

The book is very pleasant to read. In contrast to the existing literature on wavelets, there are very few theorems and proofs in this text, and the emphasis is on a clear exposition explaining the underlying ideas and situating the various techniques and ideas in an historical context. This book is recommended reading.

LIEVE DELBEKE AND WALTER VAN ASSCHE

G. H. KIROV, *Approximation with Quasi-Splines*, Institute of Physics, 1992, vii + 247 pp.

This book is a collection of results in approximation theory which, as the author points out in the preface, closely follows his research interests. It serves two purposes: first, to introduce the reader to the properties and applications of quasi-splines, and second, to survey relevant optimal recovery results which provide the motivation for the development of quasi-splines.

Interest in quasi-splines appears to be confined to the author and his colleagues. The quasi-spline of order r generated by the information $f^{(k)}(x_i), i = 1, 2, \ldots, n;\ k = 0, 1, \ldots, r$ for the function f is

$$\Phi(x) = \sum_{i=1}^{n} \sum_{k=0}^{r} k_i(x) \frac{f^{(k)}(x_i)}{k!} (x - x_i)^k, \qquad x \in [0,1],$$

where the $k_i(x)$ are a set of fundamental or basic quasi-splines that satisfy

$$\sum_{i=1}^{n} k_i(x) = 1, \qquad x \in [0,1].$$

Spline functions are the solution of many extremal problems, based on information $f(x_i)$ at nodes or mesh points x_i, $i = 1, 2, \ldots, n$. Quasi-splines are developed as a generalization of spline functions. These generalized functions may be discontinuous at the nodes. Classical splines are a special (continuous) case when $r = 0$.

The text contains seven chapters. The first is an introduction, providing required definitions of terms and notation, and also including a survey of results that follow in the remaining chapters. This survey gives brief explanations that to some extent tie the various strands of work together. Chapters 2–6 provide detailed accounts of proofs of theorems and associated results in optimal recovery theory. Kirov traces the historical development of optimal recovery results for functions in various function classes. Quasi-splines have a structure that is convenient for expressing recovery methods in these classes. The results concentrate on recovery of functions and integrals of functions of one and two variables, and include best methods of recovery in uniform and integral metrics, error bounds for best methods, and comparisons of recovery on different uniform meshes. Tables of optimal quadrature and cubature formulae are included. In the last chapter, the atomar function

$$\lambda(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\cos(tx)\prod_{k=1}^{\infty}\frac{\sin(t/2^k)}{t/2^k}\,dt,$$

which has been applied in solving boundary value problems, is developed as an application of quasi-splines.

The book is written for the approximation theorist, and in particular the specialist in optimal recovery problems. It serves a useful purpose in that much of the material it deals with has not previously appeared in English. Quasi-splines and their basic properties provide an interesting and accessible topic of study for undergraduate level mathematics students. On the other hand, the optimal recovery theorems, which make up a substantial part of the book, are more difficult to study in detail for those without expertise in the field. A number of factors contribute to this difficulty. First, the book is essentially a collection of research papers with limited additional discussion. Whilst Kirov provides an account of the important results pertaining to his particular range of interests as an approximation theorist, he does not attempt to place the work into the broad spectrum of recent developments in optimal recovery methods. Second, access to cited work, almost exclusively in Russian and Bulgarian, appears to be required for some details to be clarified, and third, complicated notation makes reading difficult.

ROBERT CHAMPION

S. THANGAVELU, *Lectures on Hermite and Laguerre Expansions*, Mathematical Notes, Vol. **24**, Princeton University Press, 1993, xv + 195 pp.

The Hermite polynomials H_k and the Laguerre polynomials L_k^α are among the basic special functions of mathematical analysis. When multiplied by their respective weight functions $e^{-x^2/2}$ and $e^{-x/2}x^{\alpha/2}$ and suitably normalized, they become the Hermite functions h_k and Laguerre functions l_k^α, which are orthonormal bases for $L^2(\mathbb{R})$ and $L^2(0,\infty)$, respectively. The Hermite functions have a natural generalization to $\mathbb{R}^n$, namely, $h_\mu(x) = \prod_1^n h_{\mu_j}(x_j)$, which again form an orthonormal basis for $L^2(\mathbb{R}^n)$.

The object of this monograph is a detailed study of the "Fourier analysis" of these bases, encompassing the sorts of problems that have long been studied for Fourier series. For instance, if $f \in L^p$, does the Hermite or Laguerre expansion of f converge to f in the L^p norm? What happens when these expansions are interpreted in terms of summability methods? If T is an operator on L^2 that is diagonal with respect to the Hermite or Laguerre basis, what conditions on the eigenvalues will guarantee that T is well-behaved on other function spaces? Interest in these questions has increased greatly in recent years because of the connection of Hermite and Laguerre functions with the representations of the Heisenberg group, which provides not only motivation for their study but tools for their solution.